FINANCIAL MATH 101
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COMPOUND INTEREST

Define…

FV	Future Value
PV	Present Value
y	years
p	compounding periods per year (365 days, 52 weeks, 12 months, 4 quarters)
N	Number of periods (y • p)
I	annual *nominal* Interest rate
I/p	*periodic* interest rate
E	annual Effective interest rate

Formulas…

Years	FV
1	$PV \bullet (1+I)$
2	$PV \bullet (1+I) \bullet (1+I) = PV \bullet (1+I)^2$
…	
y	$PV \bullet (1+I)^y$

In general, with p compounding periods per year:

$$FV = PV \bullet (1+I/p)^N \quad (1)$$

Example: You give Harvey CORE $10k and he promises to compound monthly at nominal interest rate 10% for five years. How much do you get back in five years?

We have PV=$10,000, I=0.1, p=12, y=5, giving N=60 and FV=$16,453.

And if *continuous* compounding (where e is base of natural logarithms, 2.718…):

$$FV = PV \bullet e^{yI} \quad (2)$$

Example: The Captain does Harvey one better (as usual). She will compound continuously, even as we speak. How much does she give back?

We have PV=$10,000, y=5, I=0.1, giving FV=$16,487, or $34 more than Harvey!

Re-expressing (1):

$PV = FV / (1+I/p)^N$	(3)

Same formula to deflate future $ to current $ based on inflation rate I

Example: Given FV in Example (1), all else the same, what's PV? PV=$10,000.

Example: Harvey's feeling good and says he will fork over a cool $1 million 20 years from now, no strings (so he says). The annualized inflation rate during the 1990s was about 3%. If that inflation rate holds over the next 20 years, what would Harvey's generosity be worth today?

We have FV=$1,000,000, I=0.03, p=1, y=20 (so N=20), giving PV=$553,676.

Annual effective interest rate:

FV based on effective interest rate ≡ FV based on periodic interest rate

$PV \bullet (1+E)^y = PV \bullet (1+I/p)^N$

~~PV~~ $\bullet (1+E)^y =$ ~~PV~~ $\bullet (1+I/p)^{y \bullet p}$

Taking yth root:

$(1+E) = (1+I/p)^p$

Note that E=I when p=1 (annual compounding)

$E = (1+I/p)^p - 1$	(4)

Example: Given the data in Example (1), we have I=0.1, p=12, giving E=0.1047, or an effective rate of 10.47% compared to a nominal rate of 10%.

ANNUITIES

Set of equal *periodic* payments. Not just annual, but originally conceived as annual, thus the name *annu*ity. The most common period is monthly, as in mortgages and consumer loans.

Define…

PMT Payment (Annuity)

Formulas…

$FV = PMT + PMT \bullet (1+I/p)^1 + PMT \bullet (1+I/p)^2 + \ldots + PMT \bullet (1+I/p)^N$

Last payment

First payment, same as formula (1)

This is the sum of a *geometric series*:

$FV = PMT \bullet [(1+I/p)^N - 1] \,/\, (I/p)$	(5)

Plug (5) into (3) to get after some algebraic manipulations:

$PV = PMT \bullet [1-(1+I/p)^{-N}] \,/\, (I/p)$	(6)

Or solving for PMT:

PV is the amount of the loan and PMT is the payment in loan problems.

$PMT = PV \bullet (I/p) \, / \, [1-(1+I/p)^{-N}]$	(7)

Example: What's the monthly payment on a $250,000 house loan over 30 years at 7%?

Given PV=$250,000, I=0.07, p=12, y=30, we have N=360 and PMT=$1663.26.

ANNUALIZED RETURN

Define…

r_j return in year j

A Annualized return (average return per year)

Formulas…

$FV = PV \bullet (1 + r_1) \bullet (1 + r_2) \bullet \ldots \bullet (1 + r_y)$

Plug formula (1) for FV, noting that A is used for I/p:

~~PV~~ $\bullet (1 + A)^y =$ ~~PV~~ $\bullet (1 + r_1) \bullet (1 + r_2) \bullet \ldots \bullet (1 + r_y)$

This is a geometric mean

Solving for A:

$A = [\prod(1 + r_j)]^{1/y} - 1$ (j=1,…,y)	(8)

Example: Returns are **20%** and **5%** over two years. The annualized return is:

$A = [(1.20) \bullet (1.05)]^{1/2} - 1 = [1.26]^{1/2} - 1 = 1.122 – 1 = 0.122$ or **12.2%** (not the arithmetic mean **12.5%**).